BIPOLAR INTERVAL VALUED INTUITIONISTIC FUZZY NECESSITY OPERATOR

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Abstract

In this paper we have introduced the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified its property.

Keyword

Bipolar Interval Valued Intuitionistic Fuzzy Topological Space, Bipolar Interval Valued Intuitionistic Fuzzy Set.

1. Introduction:

Lee introduced the concept of Bipolar fuzzy set. In Bipolar Intuitionistic Fuzzy Topology the membership and non-membership degree of the fuzzy set lies in the range [0,1] and [-1,0][21]. In this paper we have introduced the Bipolar Interval Valued Intuitionistic Fuzzy necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified that the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topological space.

2. Definition:

Let X be a non-empty set, and let A be a Bipolar interval valued intuitionistic fuzzy set on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X), then the necessity operator on A is defined as

i. $$[\ ]A = \left\{ \left\langle \begin{matrix} x, \left[\mu^P_{AL}(x), \mu^P_{AU}(x)\right], \left[1-\mu^P_{AU}(x), 1-\mu^P_{AL}(x)\right], \\ \left[\mu^N_{AL}(x), \mu^N_{AU}(x)\right], \left[-1+\mu^N_{AU}(x), -1+\mu^N_{AL}(x)\right] \end{matrix} \right\rangle \mid x \in X \right\}$$

2.1. Theorem:

Let (X,τ) be a Bipolar Interval Valued Intuitionistic Fuzzy Topological Space (BIVIFTS). Based on the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy set A on X, we can also construct several BIVIFTSs on X as

$$\tau_N = \{[\ ]A \mid A \in \tau\}$$

i.e., the necessity operator defined in the above definition itself forms a topology.

Proof:

In order to prove the topology we have to prove the following

Let S be a set and τ be a family of bipolar interval valued intuitionistic fuzzy subset of S. The family τ is called a Bipolar Interval Valued Intuitionistic Fuzzy Topology (BIVIFT) on S if τ satisfies the following axioms

i. $0_s, 1_s \in \tau$

ii. If $\{A_i; i \in I\} \subseteq \tau$, *then* $\bigcup_{i=1}^{\infty} A_i \in \tau$

iii. If $A_1, A_2, A_3 \ldots A_n \in \tau$, *then* $\bigcap_{i=1}^{n} A_i \in \tau$

Let $A_1, A_2, \ldots A_i$ be Bipolar interval valued intuitionistic fuzzy subsets on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X).

To prove necessity operator is a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X)

i. obviously $0_s, 1_s \in \tau_N$

ii.

$$A \cup B = \left\{ \left\langle \begin{array}{l} \left[x, \mu^{P}_{(A\cup B)L}(x), \mu^{P}_{(A\cup B)U}(x)\right], \left[\mu^{N}_{(A\cup B)L}(x), \mu^{N}_{(A\cup B)U}(x)\right], \\ \left[\nu^{P}_{(A\cup B)L}(x), \nu^{P}_{(A\cup B)U}(x)\right], \left[\nu^{N}_{(A\cup B)L}(x), \nu^{N}_{(A\cup B)U}(x)\right] \end{array} \right\rangle \mid x \in X \right\}$$

where

$$\mu^{P}_{(A\cup B)L}(x) = \min\left\{\mu^{P}_{AL}(x), \mu^{P}_{BL}(x)\right\}$$

$$\mu^{p}_{(A\square B)U}(x)=\max\left\{\mu^{p}_{AU}(x),\mu^{p}_{BU}(x)\right\}$$
$$\mu^{N}_{(A\square B)L}(x)=\max\left\{\mu^{N}_{AL}(x),\mu^{N}_{BL}(x)\right\}$$

$$\mu^{N}_{(A\square B)U}(x)=\min\left\{\mu^{N}_{AU}(x),\mu^{N}_{BU}(x)\right\}$$

$$\mu^{p}_{(A\square B)L}(x)=\min\left\{\mu^{p}_{AL}(x),\mu^{p}_{BL}(x)\right\}$$

$$\mu^{p}_{(A\square B)U}(x)=\max\left\{\mu^{p}_{AU}(x),\mu^{p}_{BU}(x)\right\}$$
$$\mu^{N}_{(A\square B)L}(x)=\max\left\{\mu^{N}_{AL}(x),\mu^{N}_{BL}(x)\right\}$$

$$\mu^{N}_{(A\square B)U}(x)=\min\left\{\mu^{N}_{AU}(x),\mu^{N}_{BU}(x)\right\}$$

$$\square[\ ]A_1\square[\ ]A_2=\left\{\left\langle \begin{array}{l}\left[x,\mu^{p}_{([\ ]A_1\square[\ ]A_2)L}(x),\mu^{p}_{([\ ]A_1\square[\ ]A_2)U}(x)\right],\\ \left[\mu^{N}_{([\ ]A_1\square[\ ]A_2)L}(x),\mu^{N}_{([\ ]A_1\square[\ ]A_2)U}(x)\right],\\ \left[\mu^{p}_{([\ ]A_1\square[\ ]A_2)L}(x),\mu^{p}_{([\ ]A_1\square[\ ]A_2)U}(x)\right],\\ \left[\mu^{N}_{([\ ]A_1\square[\ ]A_2)L}(x),\mu^{N}_{([\ ]A_1\square[\ ]A_2)U}(x)\right]\end{array}\right\rangle \mid x\in X\right\}$$

where

$$\mu^{p}_{([\ ]A_1\square[\ ]A_2)L}(x)=\min\left\{\mu^{p}_{[\ ]A_1L}(x),\mu^{p}_{[\ ]A_2L}(x)\right\}$$
$$\mu^{p}_{([\ ]A_1\square[\ ]A_2)U}(x)=\max\left\{\mu^{p}_{[\ ]A_1U}(x),\mu^{p}_{[\ ]A_2U}(x)\right\}$$
$$\mu^{N}_{([\ ]A_1\square[\ ]A_2)L}(x)=\max\left\{\mu^{N}_{[\ ]A_1L}(x),\mu^{N}_{[\ ]A_2L}(x)\right\}$$
$$\mu^{N}_{([\ ]A_1\square[\ ]A_2)U}(x)=\min\left\{\mu^{N}_{[\ ]A_1U}(x),\mu^{N}_{[\ ]A_2U}(x)\right\}$$
$$\mu^{p}_{([\ ]A_1\square[\ ]A_2)L}(x)=\min\left\{\mu^{p}_{[\ ]A_1L}(x),\mu^{p}_{[\ ]A_2L}(x)\right\}$$
$$\mu^{p}_{([\ ]A_1\square[\ ]A_2)U}(x)=\max\left\{\mu^{p}_{[\ ]A_1U}(x),\mu^{p}_{[\ ]A_2U}(x)\right\}$$
$$\mu^{N}_{([\ ]A_1\square[\ ]A_2)L}(x)=\max\left\{\mu^{N}_{[\ ]A_1L}(x),\mu^{N}_{[\ ]A_2L}(x)\right\}$$
$$\mu^{N}_{([\ ]A_1\square[\ ]A_2)U}(x)=\min\left\{\mu^{N}_{[\ ]A_1U}(x),\mu^{N}_{[\ ]A_2U}(x)\right\}$$

then

$$\mu^{p}_{([\ ]A_1\square[\ ]A_2)L}(x)=\min\left\{\mu^{p}_{A_1L}(x),\mu^{p}_{A_2L}(x)\right\}$$
$$\mu^{p}_{([\ ]A_1\square[\ ]A_2)U}(x)=\max\left\{\mu^{p}_{A_1U}(x),\mu^{p}_{A_2U}(x)\right\}$$
$$\mu^{N}_{([\ ]A_1\square[\ ]A_2)L}(x)=\max\left\{\mu^{N}_{A_1L}(x),\mu^{N}_{A_2L}(x)\right\}$$

$$\square^{N}_{([\,]A_1 \square [\,]A_2)U}(x) = \min\left\{\square^{N}_{A_1U}(x), \square^{N}_{A_2U}(x)\right\}$$

$$1 \square \square^{p}_{([\,]A_1 \square [\,]A_2)L}(x) = \min\left\{1 \square \square^{p}_{A_1L}(x), 1 \square \square^{p}_{A_2L}(x)\right\}$$

$$1 \square \square^{p}_{([\,]A_1 \square [\,]A_2)U}(x) = \max\left\{1 \square \square^{p}_{A_1U}(x), 1 \square \square^{p}_{A_2U}(x)\right\}$$

$$1 \square \square^{N}_{([\,]A_1 \square [\,]A_2)L}(x) = \max\left\{1 \square \square^{N}_{A_1L}(x), 1 \square \square^{N}_{A_2L}(x)\right\}$$

$$1 \square \square^{N}_{([\,]A_1 \square [\,]A_2)U}(x) = \min\left\{1 \square \square^{N}_{A_1U}(x), 1 \square \square^{N}_{A_2U}(x)\right\}$$

$$\square [\,]A_1 \square [\,]A_2 = \square\left\langle \begin{array}{l} \left[x, \square^{p}_{([\,]A_1 \square [\,]A_2)L}(x), \square^{p}_{([\,]A_1 \square [\,]A_2)U}(x)\right], \\ \left[\square^{N}_{([\,]A_1 \square [\,]A_2)L}(x), \square^{N}_{([\,]A_1 \square [\,]A_2)U}(x)\right], \\ \left[1 \square \square^{p}_{([\,]A_1 \square [\,]A_2)L}(x), 1 \square \square^{p}_{([\,]A_1 \square [\,]A_2)U}(x)\right], \\ \left[1 \square \square^{N}_{([\,]A_1 \square [\,]A_2)L}(x), 1 \square \square^{N}_{([\,]A_1 \square [\,]A_2)U}(x)\right] \end{array} \right\rangle \mid x \square X \square_N$$

$$\square [\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i = \square\left\langle \begin{array}{l} x, \left[\square^{p}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)L}(x), \square^{p}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)U}(x)\right], \\ \left[\square^{N}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)L}(x), \square^{N}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)U}(x)\right], \\ \left[\square^{p}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)L}(x), \square^{p}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)U}(x)\right], \\ \left[\square^{N}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)L}(x), \square^{N}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)U}(x)\right] \end{array} \right\rangle \mid x \square X \square$$

where

$$\square^{p}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)L}(x) = \min\left\{\square^{p}_{[\,]A_1L}(x), \square^{p}_{[\,]A_2L}(x), \ldots, \square^{p}_{[\,]A_iL}(x)\right\}$$

$$\square^{p}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)U}(x) = \max\left\{\square^{p}_{[\,]A_1U}(x), \square^{p}_{[\,]A_2U}(x), \ldots, \square^{p}_{[\,]A_iU}(x)\right\}$$

$$\square^{N}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)L}(x) = \max\left\{\square^{N}_{[\,]A_1L}(x), \square^{N}_{[\,]A_2L}(x), \ldots, \square^{N}_{[\,]A_iL}(x)\right\}$$

$$\square^{N}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)U}(x) = \min\left\{\square^{N}_{[\,]A_1U}(x), \square^{N}_{[\,]A_2U}(x), \ldots, \square^{N}_{[\,]A_iU}(x)\right\}$$

$$\square^{p}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)L}(x) = \min\left\{\square^{p}_{[\,]A_1L}(x), \square^{p}_{[\,]A_2L}(x), \ldots, \square^{p}_{[\,]A_iL}(x)\right\}$$

$$\square^{p}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)U}(x) = \max\left\{\square^{p}_{[\,]A_1U}(x), \square^{p}_{[\,]A_2U}(x), \ldots, \square^{p}_{[\,]A_iU}(x)\right\}$$

$$\square^{N}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)L}(x) = \max\left\{\square^{N}_{[\,]A_1L}(x), \square^{N}_{[\,]A_2L}(x), \ldots, \square^{N}_{[\,]A_iL}(x)\right\}$$

$$\square^{N}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)U}(x) = \min\left\{\square^{N}_{[\,]A_1U}(x), \square^{N}_{[\,]A_2U}(x), \ldots, \square^{N}_{[\,]A_iU}(x)\right\}$$

then

$$\square^{p}_{([\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i)L}(x) = \min\left\{\square^{p}_{A_1L}(x), \square^{p}_{A_2L}(x), \ldots, \square^{p}_{A_iL}(x)\right\}$$

$$\mu^{P}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x) = \max\left\{\mu^{P}_{A_1U}(x), \mu^{P}_{A_2U}(x), \ldots, \mu^{P}_{A_iU}(x)\right\}$$

$$\mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x) = \max\left\{\mu^{N}_{A_1L}(x), \mu^{N}_{A_2L}(x), \ldots, \mu^{N}_{A_iL}(x)\right\}$$

$$\mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x) = \min\left\{\mu^{N}_{A_1U}(x), \mu^{N}_{A_2U}(x), \ldots, \mu^{N}_{A_iU}(x)\right\}$$

$$1 - \mu^{P}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x) = \min\left\{1 - \mu^{P}_{A_1L}(x), 1 - \mu^{P}_{A_2L}(x), \ldots 1 - \mu^{P}_{A_iL}(x)\right\}$$

$$1 - \mu^{P}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x) = \max\left\{1 - \mu^{P}_{A_1U}(x), 1 - \mu^{P}_{A_2U}(x), \ldots 1 - \mu^{P}_{A_iU}(x)\right\}$$

$$1 - \mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x) = \max\left\{1 - \mu^{N}_{A_1L}(x), 1 - \mu^{N}_{A_2L}(x), \ldots 1 - \mu^{N}_{A_iL}(x)\right\}$$

$$1 - \mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x) = \min\left\{1 - \mu^{N}_{A_1U}(x), 1 - \mu^{N}_{A_2U}(x), \ldots 1 - \mu^{N}_{A_iU}(x)\right\}$$

$$\cup [\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i = \left\{\left\langle \begin{array}{l} x, \left[\mu^{P}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x), \mu^{P}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x)\right], \\ \left[\mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x), \mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x)\right], \\ \left[1 - \mu^{P}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x), 1 - \mu^{P}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x)\right], \\ \left[1 - \mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)L}(x), 1 - \mu^{N}_{([\,]A_1 \cup [\,]A_2 \cup \ldots [\,]A_i)U}(x)\right] \end{array} \right\rangle \mid x \in X \right\}_{N}$$

Hence the arbitrary union of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ.

iii.

$$A \cap B = \left\{\left\langle \begin{array}{l} x, \left[\mu^{P}_{(A\cap B)L}(x), \mu^{P}_{(A\cap B)U}(x)\right], \left[\mu^{N}_{(A\cap B)L}(x), \mu^{N}_{(A\cap B)U}(x)\right], \\ \left[\nu^{P}_{(A\cap B)L}(x), \nu^{P}_{(A\cap B)U}(x)\right], \left[\nu^{N}_{(A\cap B)L}(x), \nu^{N}_{(A\cap B)U}(x)\right] \end{array} \right\rangle \mid x \in X \right\}$$

where

$$\mu^{P}_{(A\cap B)L}(x) = \max\left\{\mu^{P}_{AL}(x), \mu^{P}_{BL}(x)\right\}$$

$$\mu^{P}_{(A\cap B)U}(x) = \min\left\{\mu^{P}_{AU}(x), \mu^{P}_{BU}(x)\right\}$$

$$\mu^{N}_{(A\cap B)L}(x) = \min\left\{\mu^{N}_{AL}(x), \mu^{N}_{BL}(x)\right\}$$

$$\mu^{N}_{(A\cap B)U}(x) = \max\left\{\mu^{N}_{AU}(x), \mu^{N}_{BU}(x)\right\}$$

$$\nu^{P}_{(A\cap B)L}(x) = \max\left\{\nu^{P}_{AL}(x), \nu^{P}_{BL}(x)\right\}$$

$$\nu^{P}_{(A\cap B)U}(x) = \min\left\{\nu^{P}_{AU}(x), \nu^{P}_{BU}(x)\right\}$$

$$\nu^{N}_{(A\cap B)L}(x) = \min\left\{\nu^{N}_{AL}(x), \nu^{N}_{BL}(x)\right\}$$

$$\nu^N_{(A\cap B)U}(x) = \max\{\nu^N_{AU}(x), \nu^N_{BU}(x)\}$$

then

$$([\,]A_1 \cap [\,]A_2) = \left\{ \left\langle \begin{array}{l} \left[x, \mu^p_{([\,]A_1\cap[\,]A_2)L}(x), \mu^p_{([\,]A_1\cap[\,]A_2)U}(x)\right], \\ \left[\mu^N_{([\,]A_1\cap[\,]A_2)L}(x), \mu^N_{([\,]A_1\cap[\,]A_2)U}(x)\right], \\ \left[\nu^p_{([\,]A_1\cap[\,]A_2)L}(x), \nu^p_{([\,]A_1\cap[\,]A_2)U}(x)\right], \\ \left[\nu^N_{([\,]A_1\cap[\,]A_2)L}(x), \nu^N_{([\,]A_1\cap[\,]A_2)U}(x)\right] \end{array} \right\rangle \mid x \in X \right\}$$

where

$$\mu^p_{([\,]A_1\cap[\,]A_2)L}(x) = \max\{\mu^p_{[\,]A_1L}(x), \mu^p_{[\,]A_2L}(x)\}$$

$$\mu^p_{([\,]A_1\cap[\,]A_2)U}(x) = \min\{\mu^p_{[\,]A_1U}(x), \mu^p_{[\,]A_2U}(x)\}$$

$$\mu^N_{([\,]A_1\cap[\,]A_2)L}(x) = \min\{\mu^N_{[\,]A_1L}(x), \mu^N_{[\,]A_2L}(x)\}$$

$$\mu^N_{([\,]A_1\cap[\,]A_2)U}(x) = \max\{\mu^N_{[\,]A_1U}(x), \mu^N_{[\,]A_2U}(x)\}$$

$$\nu^p_{([\,]A_1\cap[\,]A_2)L}(x) = \max\{\nu^p_{[\,]A_1L}(x), \nu^p_{[\,]A_2L}(x)\}$$

$$\nu^p_{([\,]A_1\cap[\,]A_2)U}(x) = \min\{\nu^p_{[\,]A_1U}(x), \nu^p_{[\,]A_2U}(x)\}$$

$$\nu^N_{([\,]A_1\cap[\,]A_2)L}(x) = \min\{\nu^N_{[\,]A_1L}(x), \nu^N_{[\,]A_2L}(x)\}$$

$$\nu^N_{([\,]A_1\cap[\,]A_2)U}(x) = \max\{\nu^N_{[\,]A_1U}(x), \nu^N_{[\,]A_2U}(x)\}$$

then

$$\mu^p_{([\,]A_1\cap[\,]A_2)L}(x) = \max\{\mu^p_{A_1L}(x), \mu^p_{A_2L}(x)\}$$

$$\mu^p_{([\,]A_1\cap[\,]A_2)U}(x) = \min\{\mu^p_{A_1U}(x), \mu^p_{A_2U}(x)\}$$

$$\mu^N_{([\,]A_1\cap[\,]A_2)L}(x) = \min\{\mu^N_{A_1L}(x), \mu^N_{A_2L}(x)\}$$

$$\mu^N_{([\,]A_1\cap[\,]A_2)U}(x) = \max\{\mu^N_{A_1U}(x), \mu^N_{A_2U}(x)\}$$

$$1-\mu^p_{([\,]A_1\cap[\,]A_2)L}(x) = \max\{1-\mu^p_{A_1L}(x), 1-\mu^p_{A_2L}(x)\}$$

$$1-\mu^p_{([\,]A_1\cap[\,]A_2)U}(x) = \min\{1-\mu^p_{A_1U}(x), 1-\mu^p_{A_2U}(x)\}$$

$$1-\mu^N_{([\,]A_1\cap[\,]A_2)L}(x) = \min\{1-\mu^N_{A_1L}(x), 1-\mu^N_{A_2L}(x)\}$$

$$1 \square \square^{N}_{([\,]A_1 \square [\,]A_2)_U}(x) = \max\left\{1 \square \square^{N}_{A_1U}(x), 1 \square \square^{N}_{A_2U}(x)\right\}$$

$$\square[\,]A_1 \square [\,]A_2 = \left\{\left\langle \begin{array}{l} x, \left[\square^{p}_{([\,]A_1\square[\,]A_2)L}(x), \square^{p}_{([\,]A_1\square[\,]A_2)U}(x)\right],, \\ \left[\square^{N}_{([\,]A_1\square[\,]A_2)L}(x), \square^{N}_{([\,]A_1\square[\,]A_2)U}(x)\right], \\ \left[1 \square \square^{p}_{([\,]A_1\square[\,]A_2)L}(x), 1 \square \square^{p}_{([\,]A_1\square[\,]A_2)U}(x)\right],, \\ \left[1 \square \square^{N}_{([\,]A_1\square[\,]A_2)L}(x), 1 \square \square^{N}_{([\,]A_1\square[\,]A_2)U}(x)\right] \end{array} \right\rangle \mid x \square X \right\} \square \square_N$$

$$\square[\,]A_1 \square [\,]A_2 \square \ldots [\,]A_i = \left\{\left\langle \begin{array}{l} x, \left[\square^{p}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)L}(x), \square^{p}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)U}(x)\right], \\ \left[\square^{N}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)L}(x), \square^{N}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)U}(x)\right], \\ \left[\square^{p}_{([\,]A_1\square[\,]A_2\square\square\ldots[\,]A_i)L}(x), \square^{p}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)U}(x)\right], \\ \left[\square^{N}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)L}(x), \square^{N}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)U}(x)\right] \end{array} \right\rangle \mid x \square X \right\}$$

where

$$\square^{p}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)L}(x) = \max\left\{\square^{p}_{[\,]A_1L}(x), \square^{p}_{[\,]A_2L}(x), \ldots, \square^{p}_{[\,]A_iL}(x)\right\}$$

$$\square^{p}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)U}(x) = \min\left\{\square^{p}_{[\,]A_1U}(x), \square^{p}_{[\,]A_2U}(x), \ldots, \square^{p}_{[\,]A_iU}(x)\right\}$$

$$\square^{N}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)L}\, x) = \min\left\{\square^{N}_{[\,]A_1L}(x), \square^{N}_{[\,]A_2L}(x), \ldots, \square^{N}_{[\,]A_iL}(x)\right\}$$

$$\square^{N}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)U}(x) = \max\left\{\square^{N}_{[\,]A_1U}(x), \square^{N}_{[\,]A_2U}(x), \ldots, \square^{N}_{[\,]A_iU}(x)\right\}$$

$$\square^{p}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)L}(x) = \max\left\{\square^{p}_{[\,]A_1L}(x), \square^{p}_{[\,]A_2L}(x), \ldots, \square^{p}_{[\,]A_iL}(x)\right\}$$

$$\square^{p}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)U}(x) = \min\left\{\square^{p}_{[\,]A_1U}(x), \square^{p}_{[\,]A_2U}(x), \ldots, \square^{p}_{[\,]A_iU}(x)\right\}$$

$$\square^{N}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)L}(x) = \min\left\{\square^{N}_{[\,]A_1L}(x), \square^{N}_{[\,]A_2L}(x), \ldots, \square^{N}_{[\,]A_iL}(x)\right\}$$

$$\square^{N}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)U}(x) = \max\left\{\square^{N}_{[\,]A_1U}(x), \square^{N}_{[\,]A_2U}(x), \ldots, \square^{N}_{[\,]A_iU}(x)\right\}$$

then

$$\square^{p}_{([\,]A_1\square[\,]A_2\square\ldots[\,]A_i)L}(x) = \max\left\{\square^{p}_{A_1L}(x), \square^{p}_{A_2L}(x), \ldots, \square^{p}_{A_iL}(x)\right\}$$

$$\mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x) = \min\left\{\mu^{p}_{A_1 U}(x), \mu^{p}_{A_2 U}(x), \ldots, \mu^{p}_{A_i U}(x)\right\}$$

$$\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x) = \min\left\{\mu^{N}_{A_1 L}(x), \mu^{N}_{A_2 L}(x), \ldots, \mu^{N}_{A_i L}(x)\right\}$$

$$\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x) = \max\left\{\mu^{N}_{A_1 U}(x), \mu^{N}_{A_2 U}(x), \ldots, \mu^{N}_{A_i U}(x)\right\}$$

$$1-\mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x) = \max\left\{1-\mu^{p}_{A_1 L}(x), 1-\mu^{p}_{A_2 L}(x), \ldots 1-\mu^{p}_{A_i L}(x)\right\}$$

$$1-\mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x) = \min\left\{1-\mu^{p}_{A_1 U}(x), 1-\mu^{p}_{A_2 U}(x), \ldots 1-\mu^{p}_{A_i U}(x)\right\}$$

$$1-\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x) = \min\left\{1-\mu^{N}_{A_1 L}(x), 1-\mu^{N}_{A_2 L}(x), \ldots 1-\mu^{N}_{A_i L}(x)\right\}$$

$$1-\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x) = \max\left\{1-\mu^{N}_{A_1 U}(x), 1-\mu^{N}_{A_2 U}(x), \ldots 1-\mu^{N}_{A_i U}(x)\right\}$$

$$\square\left([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i\right) = \left\{\left\langle \begin{array}{l} x, \left[\mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x), \mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x)\right], \\ \left[\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x), \mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x)\right], \\ \left[1-\mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x), 1-\mu^{p}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x)\right], \\ \left[1-\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)L}(x), 1-\mu^{N}_{([\,]A_1 \cap [\,]A_2 \cap \ldots [\,]A_i)U}(x)\right] \end{array} \right\rangle \mid x \in X \right\} \in \tau_N$$

Hence the finite intersection of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ.

Hence the Bipolar Interval Valued Intuitionistic Necessity Operators itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topology τ.

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